MULTIMEDIA UNIVERSITY

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FINAL EXAMINATION

TRIMESTER 3, 2021/2022 (2135)

PEM0046 – INTRODUCTION TO PROBABILITY AND STATISTICS

(Foundation in Engineering)

3 OCTOBER 2022
9.00 a.m. - 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of **FIVE** printed pages including the cover page.
2. Attempt **ALL FOUR** questions. All questions carry equal marks and the distribution of the marks for each question is given.
3. Please write all your answers in the answer booklet provided. All necessary working steps **MUST** be shown.
4. Statistical table is provided.

Answer ALL questions [100 marks].

QUESTION 1 [25 marks]

a. Consider two identical boxes containing the same number of ringgit notes.
Each box contains a 1-ringgit note, a 2-ringgit note, and a 5-ringgit note.
If a note is chosen at random from each of the boxes, and let X denote the sum of the values of both notes obtained.

i. What are the possible values of X? [2 marks]

ii. Construct a probability distribution table of X. [5 marks]

iii. Determine the probability that a 5-ringgit note will be drawn from the second box. [2 marks]

iv. Determine the expected value of X. [2 marks]

b. X is a discrete random variable with values of -1, 0, 1 and 2.
If $\mathrm{E}(X^2) = 3\mathrm{E}(X)$ and $\mathrm{Var}\,(X) = \frac{8}{9}$,

i. calculate $\mathrm{E}(X)$. [3.5 marks]

ii. If $\mathrm{E}(X) = \frac{1}{3}$ and $\mathrm{P}(X = -1) = \mathrm{P}(X = 1) = \mathrm{P}(X = 2)$,
show that $\mathrm{P}(X = 0) = \frac{1}{2}$. [2.5 marks]

c. The lifetime of a certain brand of electric bulb, in hours, is modelled by the continuous random variable X with probability density function $f(x)$ given by

$$f(x) = \begin{cases} kb - kx, & , \ 0 \le x \le b \\ 0 & , \text{ elsewhere} \end{cases}$$

where k is a positive constant.

i. Find k in terms of b. [4 marks]

ii. If $k = \frac{2}{b^2}$, show that the mean of X in terms of b is $\frac{b}{3}$. [4 marks]

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QUESTION 2 [25 marks]

a. In a large factory, 25% of the workers are members of trade union A, 55% are members of trade union B, and 20% are not members of any trade union.

A sample of five workers was chosen from the workers of this factory.

Find the probability that this sample contains

i. exactly three members are from trade union A. [2 marks]

ii. Find the mean and standard deviation of the number of trade union A members in the sample. [3 marks]

iii. Calculate the probability that the majority of the sample's workers are members of trade union A given that exactly four of the sample's workers are union members. [5 marks]

b. Particles are emitted randomly from a radioactive substance at an average of six particles every 10 seconds.

Find the probability that

i. no emission occurs in 10 seconds. [1 mark]

ii. at least ten particles are emitted every 15 seconds. [3 marks]

iii. between six and nine particles are emitted in 15 seconds. [3 marks]

c. Past records show that the times taken by students (aged 14-15) at a school to run 100 meters (m) are normally distributed, with a mean of 14.6 seconds and a standard deviation of 1.50 seconds. What is the probability that a randomly selected student (aged 14-15) from the school will run 100 m in

i. less than 15 seconds. [2 marks]

During sports day, the school awards certificates to the top 20% of students (aged 14-15) in the 100 m race.

ii. Estimate the slowest times a student will take to run 100 m and receive a certificate (answer rounded to two decimal places). [5 marks]

iii. Will a student receive a certificate if his running time is 15 seconds? [1 mark]

Continued...

QUESTION 3 [25 marks]

a. A small company has 5 employees. The annual income (in Ringgit Malaysia) of each employee are given below:

114,560	**214,200**	**155,300**	**114,560**	**156,700**

List all the possible sample of size two (without replacement) from this population and construct the sampling distribution of the sample mean.

Then, find the sampling error for each sample. [12 marks]

b. According to a production senior engineer in a factory, the lifespans of all car batteries produced by his factory are known to be normally distributed with a mean of 40 months and a standard deviation of 5 months.

If a random sample of 50 car batteries is selected from the population, determine

i. the mean, and the standard deviation of the sample mean. [2.5 marks]

ii. the probability that the sample mean is no more than 40 months. [1.5 marks]

iii. the probability that the sample mean is between 40 and 42 months. [3.5 marks]

iv. the probability that the sample mean lies outside the range of 40 to 42 months. [1.5 marks]

c. The sample mean, $\bar{X}$ of each sample is calculated after a large number of random samples of size n are drawn from a normal distribution population with a mean of 64 and a standard deviation of 6.

If $P(\bar{X} > 62) = 0.8508$, estimate the value of n. [4 marks]

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QUESTION 4 [25 marks]

The following table give information on yearly incomes (in thousands of dollars) and the values of car (in thousands of dollars) owned by a sample of eight persons.

Yearly income	215	274	294	296	259	380	457	523
Value of the car	66	81	99	87	78	102	90	120

a. Is the value of the car owned that depends on the yearly income of a person or the yearly income that depends on the value of the car owned?

Do you expect a positive or negative relationship between these two variables?

[2 marks]

b. Find the least square regression equation for the above data. [13 marks]

c. Plot the scatter diagram on the graph paper provided and draw the least square regression line on the same graph. [4 marks]

d. Predict the value of the car owned by a person with a yearly income of $300,000 [2 marks]

e. Calculate the linear correlation coefficient and give your comment on the value obtained. [4 marks]

[Leave all your answers in 3 decimal points]

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